Photonic Bandgap Tuning of Photonic Crystals by Air Filling Fraction

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ABSTRACT

In this paper, the type of photonic crystals (PCs) which consists of air holes surrounded by silica as a background with square lattice arrangement in two dimension (2D) has been assumed. A plane wave expansion method (PWEM) is implemented to calculate the photonic band structure of this kind of PC, then the effect of changing air filling fraction is studied on the photonic band structure diagram, by changing the air hole radius (r_a) and lattice constant (a). As a result, a tuning and shifting of the photonic bandgaps (PBGs) has been obtained.

Keywords: Photonic Bandgap, Photonic Crystals, Air Filling Fraction.

(PCs)

(PWEM)   .

(r_a)

.(PBGs)   .(a)

INTRODUCTION

A photonic crystal (PC) is a periodic arrangement of dielectric or metallic materials, which is an optical analogy to a conventional crystal. A photonic crystal provides a possibility to control and manipulate the propagation of light (Qiu, 2000), by several parameters which characterize the crystal: lattice constant (a), air hole radius (r_a), air hole shape, refractive index of crystal construction, and type of lattice.

The fundamental motivation for developing PC was the creation of a new kind of dielectric waveguide that guides light by means of a two-dimensional photonic band gaps (PBGs). The original 1991 idea of photonic crystal fibers (PCF), then, was to trap light in a

hollow core by means of a two-dimensional "photonic crystal" to prevente the escape of light and avoid the need for total internal reflection (TIR). (Russell and Pearce, 2010).

Photonic crystals are designed to exhibit interesting properties in the propagation of electromagnetic waves. These crystals today play one of the most important roles in telecommunication technique due to their unique possibility to prohibit light propagation inside some frequency range due to the existence of PBGs. The most popular applications of PCs are photonic crystal fibers, photonic crystal lasers, waveguides, and filters (Johnson and Joannopoulos, 2003).

There is a relation between the wave vectors and the frequencies when electromagnetic waves propagate in a photonic crystal, which is known as a dispersion relation. This relation leads to the band structure of the photonic crystal, which is known as photonic band gap structures, It gives information about the propagation properties of electromagnetic (EM) radiation within the photonic crystal (Joannopoulos *et al.*, 2009), (Skorobogatiy and Yang, 2009).

The photonic band gap structure is the main feature used in designing such types of devices, and their analysis has been discussed by different techniques such as transfer matrix method (Cao *et al.*, 2004), finite difference frequency domain method (Yu and Chang, 2004), finite element method (Andonegui and Angel, 2012) and plane wave expansion method (Bin *et al.*, 2011), (Salcedo-Reyes, 2012). Each method has its own advantages and disadvantages to simulate specific problems related to PBGs structure.

The main computation technique which is used to obtain such a feature is the plane wave expansion (PWE) method. This method is fruitful in studying band gap and filed mode, besides it is simple enough to be easily implemented.

In this research, photonic band structure has been obtained, then different photonic band structures for different air hole radius and lattice constant are compared.

THEORETICAL DETAILS

Plane Wave Expansion Method :

In order to calculate the electromagnetic frequency spectrum of these dielectric structures, the computational techniques described by Skorobogatiy and Yang are employed. The Maxwell's equations can be rearranged to yield the eigenvalue equation (Joannopoulos *et al.*, 2009), (Skorobogatiy and Yang, 2009) :

$$\nabla \times \left[\frac{1}{\varepsilon_r(\vec{r})} \nabla \times \bar{H} \right] = \frac{\omega^2}{c^2} \bar{H} \qquad (1)$$

Where $\varepsilon_r(\vec{r})$ is the periodic dielectric constant function of photonic crystals, ω is the angular frequency, $c = 1/\sqrt{\varepsilon_0 \mu_o}$ is the speed of light, and μ_o is the material permeability. The magnetic field $H(\vec{r})$ can be expressed as a sum of plane waves (Guo, 2003):

$$H(\bar{r}) = \sum_{\bar{G}} \sum_{\lambda=1,2} h_{G\lambda} \bar{e}_\lambda e^{i(\bar{k}+\bar{G}).\bar{r}} \qquad (2)$$

Where $\vec{k}$ is a wave vector in the Brillouin zone of the lattice, $\vec{G}$ is a reciprocal lattice vector, and $\hat{e}_\lambda$ are unit vectors perpendicular to $\vec{k} + \vec{G}$, because of the transverse nature of **H** (i.e.,

$\mathbf{\nabla \times H = 0}$). Substituting into equation (1), we obtain the following matrix equations (Guo, 2003):

$$\sum_{G} |\vec{k} + \vec{G}||\vec{k} + \vec{G}'| \, \varepsilon_r^{-1}\left(\vec{G} - \vec{G}'\right) \begin{pmatrix} \hat{e}_2 . \hat{e}'_2 & -\hat{e}_2 . \hat{e}'_1 \\ -\hat{e}_1 . \hat{e}'_2 & \hat{e}_1 . \hat{e}'_1 \end{pmatrix} h_{G'\lambda'} = \left(\frac{\omega}{c}\right)^2 h_{G\lambda} \qquad (3)$$

Equation (3) can be solved for different wave vectors $\vec{k}$, we can obtain a series of eigen-frequencies ω, which composes the band structures of photonic crystals, this band structure resulted by plotting the dispersion relation along a characteristic path along the edges of the irreducible Brillouin zone (IBZ) which is the smallest section of the full Brillouin zone that is bounded by symmetry lines of the Bravaies lattice to which the Brillouin zone is attached.

CALCULATIONS AND RESULTS

Air filling fraction is defined as the ratio of the volume of all holes to the whole space volume. This ratio has different values in each lattice geometry, and in square lattice with circular hole cylinders as given by (Guo, 2003):

$$f = \frac{\pi r_a^2}{a^2} \qquad (4)$$

Therefore, to investigate its effect on photonic bandgaps, the photonic band structure is evaluated and plotted as a function of hole radius (r_a) and lattice constant (a).

1. PBG Tuning by Hole Radius

In order to calculate the photonic band structure of a special structure with periodic arrangement, the plane wave expansion method may still be applied where the smallest region describing the periodic structure (unit cell) is enlarged to include the so-called "supercell approximation". That is, we imagine a cavity or waveguide structure that is periodically repeated at large intervals in space (Joannopoulos *et al.*, 2009). Example of the unit cell (bold square) and the 2x2 supercell shown in the left of (Fig. 1). Lattice geometry (square lattice, triangular lattice, honeycomb lattice) with supercell approximation are important factors to determine the photonic band gap and other optical properties of photonic crystal.

Then, we investigate the effect of air filling fraction change on PBG in 2D structure with 2x2 supercell by evaluating the photonic band structure of different air hole radius. This photonic crystal has 2D square lattice structure (the two basis vectors are orthogonal) and circular air hole shape. Note that the circular shape of air hole is known as "cell" with dielectric constant $\varepsilon_a = 1$, surrounded by silica background which has the dielectric constant $\varepsilon_b = 2.14$ in optical region. Finally, the photonic band structure is evaluated in irreducible Brillouin zone (IBZ), which has high symmetry points highlighted : Γ, X, and M in (Fig. 1).
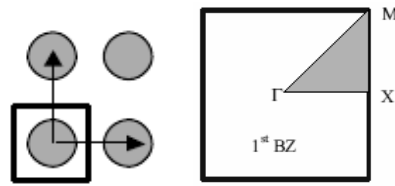
Fig. 1: **2D square lattice with 2x2 supercell, Left: Unit cell and basis lattice vectors, Right :the 1st Brillouin zone and the irreducible Brillouin zone IBZ (Guo, 2003).**

The result of square lattice PC which has air hole radius: $r_a = 0.05\,a$ is shown in (Fig. 2) for TM modes. The results of the same geometrical lattice PC, but with different radius of air hole : $r_a = 0.1\,a$, $r_a = 0.2\,a$, $r_a = 0.3\,a$, and $r_a = 0.4\,a$, all for TM modes, are plotted in figures Fig.3, 4, 5, and 6, respectively.
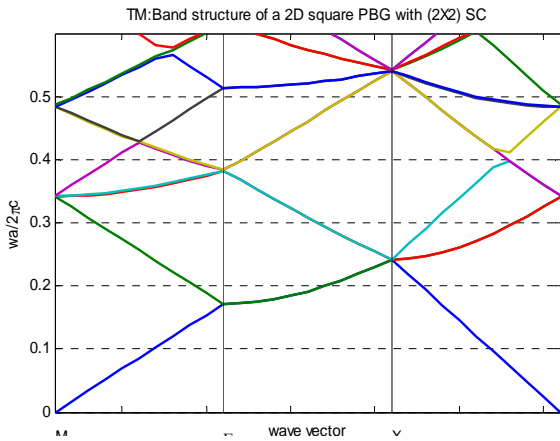
Fig. 2: Photonic band structures of PC, with air hole radius $r_a = 0.05\,a$ in silica background.
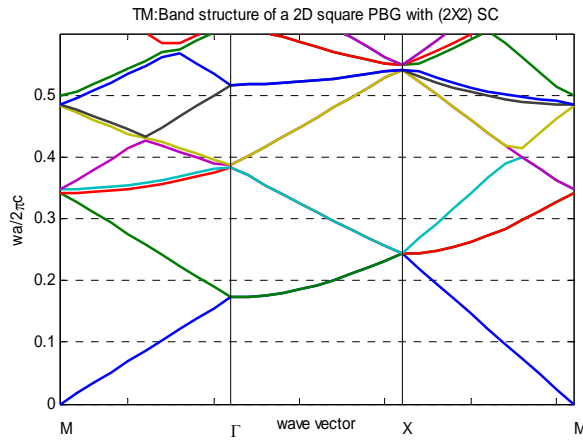
Fig. 3: Photonic band structures of PC, with air hole radius $r_a = 0.1\,a$ in silica background.
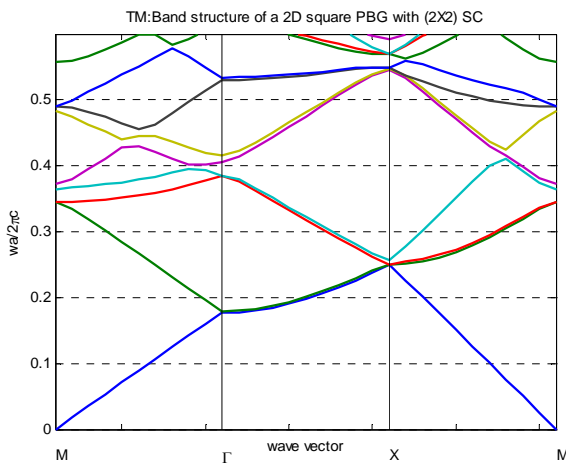

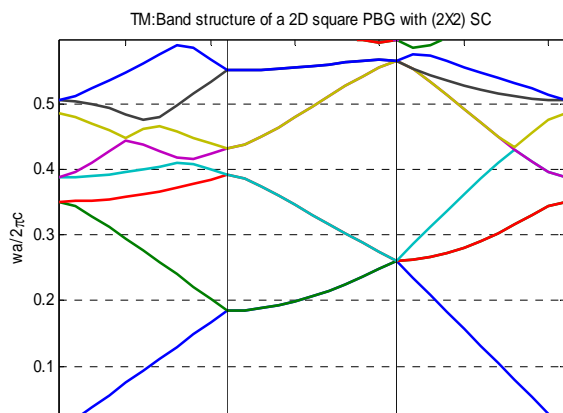**Fig. 4: Photonic band structures of PC, with air hole radius $r_a = 0.2\,a$ in silica background.**

Fig. 5: Photonic band structures of PC, with air hole radius $r_a = 0.3\,a$ in silica background.
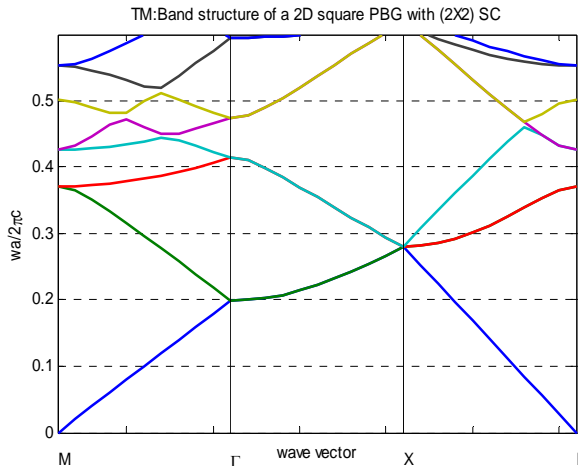
Fig. 6 : Photonic band structures of PC, with air hole radius $r_a = 0.4\,a$ in silica background.

The bandwidth between the second and the third band was determined in the (Γ→X) wave vector direction (Fig. 7) to show how we can tune the bandwidth range of 2D square lattice structure using different values of hole radius in this wave direction.

2. PBG Tuning by Lattice Constant :

We investigate the effect of air filling fraction change on PBG in 2D structure with 2x2 supercell by evaluate the photonic band structure of different lattice constant (lattice spacing). The bandwidth between the same bands has been determined in the same wave vector direction, and different values of lattice constant. (Fig. 8) shows how we can tune the bandwidth range of 2D square lattice structure using changing hole radius. All structures have the same air hole radius $r_a = 0.4\,a$.
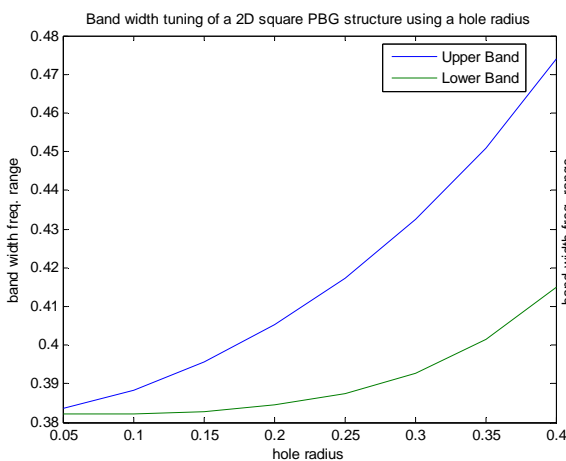

Fig. 7: Bandwidth tuning of 2D square PBG structure using hole radius change in (Γ→X) wave vector direction.

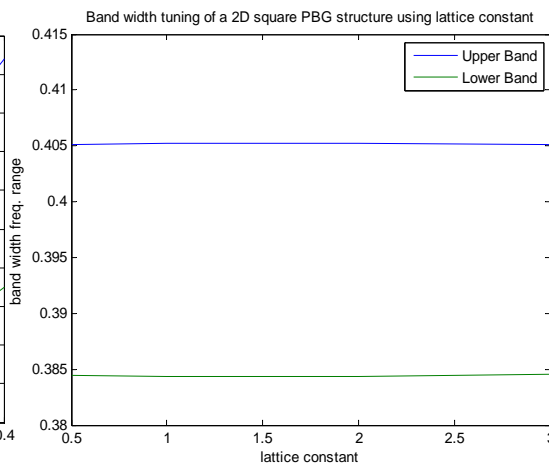Fig. 8 : Bandwidth tuning of 2D square PBG structure using lattice constant changing in (Γ→X) wave vector direction.

DISCUSSION

The results of photonic band structure diagrams are obtained by using plane wave expansion method. This work shows that air filling fraction is structural parameter affects on photonic band gaps of photonic crystal, which can be accounted as a result of changing silica percentage in the whole structure. This is due to the effective refractive index shift closer or farther away from the conventional value of the refractive index of pure silica.

It is evident that air filling fraction can tune and modulate the photonic bandgap by air hole radius change of these classes of geometry in some wave vector directions, and more clearly that the bandwidth is flexible and efficient by the radius of air holes, and have range increases exponentially while the radius increases in $(\Gamma \rightarrow X)$ wave vector direction.

The photonic bandgaps increase in frequency as $\frac{r_a}{a}$ increase, since the frequency scales as $\frac{1}{\sqrt{\varepsilon}}$ in a medium of dielectric constant ε and, as $\frac{r_a}{a}$ increase, the average dielectric constant of the medium steadily decreases as the air holes grow. This is an expected feature by (Joannopoulos *et al.*, 2009) for "square" lattice of air columns (holes) in dielectric medium.

The photonic band gap variation becomes less important when lattice constant changes, as a result of being denominator of air filling fraction, as well as the quadrature located in the ratio; Furthermore, there are limitations in changing the lattice constant with the dimensions of the structure as a whole, in addition to the difficulty of approaching air holes from each other very much.

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